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COORDINATE TRANSFORMATION USING THE LEAST SQUARES METHOD: A CASE STUDY ON TRABZON, TURKEY

TRANSFORMACIJA KOORDINATA METODOM NAJMANJIH KVADRATA: STUDIJA SLUČAJA U TRABZONU, TURSKA

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ABSTRACT

The ED50 datum uses the Hayford ellipsoid and the Greenwich meridian as the prime meridian. In Turkey, the ED50 datum with the Hayford ellipsoid was used until 2001, after which the ITRF datum with the GRS80 ellipsoid became the standard. To utilize coordinate systems across different datums, these systems must be associated and transformed accordingly. This study determines 2D transformation parameters for converting the ED50 datum to the ITRF-96 datum.

For this transformation, common points in both the ED50 and ITRF systems were selected in the Maçka and Ortahisar districts of Trabzon province. The two-dimensional similarity (Helmert) and Affine transformation methods were applied to these common points to establish the relationship between the two systems within the study area. Statistical data was gathered to evaluate the performance of the transformation parameters and process. Additionally, test points within the study areas were selected, and their ED50 datum coordinates were transformed to ITRF-96 coordinates using these parameters. The differences between the transformed coordinates and the current coordinates of these test points were analyzed, with root mean square error (RMSE), minimum and maximum

SAŽETAK

ED50 datum temelji se na Hayfordovom elipsoidu i početnom meridijanu koji prolazi kroz Greenwich. U Turskoj se datum ED50 s Hayfordovim elipsoidom koristio do 2001. godine, nakon čega je usvojen datum ITRF s GRS80 elipsoidom. Za primjenu koordinatnih sistema u različitim datumima, ti sistemi moraju biti povezani i transformisani pomoću odgovarajućih parametara. Ova studija se bavi određivanjem parametara 2D transformacije između datuma ED50 i ITRF-96.

Za transformaciju su odabrane zajedničke tačke u sistemima ED50 i ITRF u okruzima Maçka i Ortahisar, koji se nalaze u pokrajini Trabzon. Pomoću njih primijenjene su metode dvodimenzionalne slične (Helmert) i afine transformacije kako bi se definisao odnos između dva sistema unutar područja istraživanja. Prikupljeni su statistički podaci kako bi se procijenila tačnost transformacionih parametara i učinkovitost cijelog postupka. Nadalje, unutar područja istraživanja odabrane su kontrolne tačke čije su ED50 koordinate transformisane u ITRF-96 koordinate pomoću utvrđenih parametara. Analizirane su razlike između transformisanih i stvarnih koordinata ovih testnih tačaka, uključujući srednju kvadratnu grešku (RMSE),

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errors, and mean absolute error (MAE) values calculated.

In this study, a total of 197 points were used in Değirmendere, and 106 points were used in Maçka. It was observed that the sensitivity parameters of the transformation in the Maçka region are better than those in Değirmendere, potentially due to the number of common points. When comparing the Affine and Similarity transformations, the similarity transformation yielded significantly better results. All calculations were conducted using MATLAB R2022b software.

Keywords: ED50, ITRF-96, 2D coordinate transformations, Helmert transformation, Affine transformation

minimalne i maksimalne greške te srednje apsolutne greške (MAE).

U istraživanju je korišteno ukupno 197 tačaka u regiji Değirmendere i 106 točaka u regiji Maçka. Uočeno je da su pokazatelji tačnosti transformacije u regiji Maçka bolji nego u Değirmendereu, što se može pripisati većem broju zajedničkih tačaka. Poređenjem metoda afine i slične transformacije, slična transformacija dala je značajno bolje rezultate. Svi izračuni provedeni su korištenjem softvera MATLAB R2022b.

Ključne riječi: ED50, ITRF-96, 2D transformacije koordinata, Helmertova transformacija, Afina transformacija

1 INTRODUCTION

The ED50 datum uses the Hayford ellipsoid and the Greenwich meridian as the prime meridian. Toward the 2000s, due to technological advancements, Turkey's tectonic structure, and other factors, the accuracy provided by the existing country datum or coordinates no longer met expectations for geodetic studies. This led to the need for a new coordinate system, resulting in the establishment of the Turkish National Basic GPS Network (TUTGA) based on Global Positioning System technology. Later, this network evolved into the Continuously Operating Reference Stations (CORS-TR) system, forming a national coordinate framework based on the ITRF coordinate system.

A coordinate system is a location system used to specify the position of any point on Earth. Coordinate systems are defined by establishing their origin points and coordinate axes. The process of transferring coordinates from one coordinate system to another is called as coordinate transformation. Coordinate transformation is carried out to enable the use of points with coordinates determined in different datums together. When it is required to determine the equivalent of coordinate data from different coordinate systems in a common coordinate system, a datum transformation is applied. A datum can be defined as fixed information about the reference surface, which is accepted as the basis for geodetic calculations.

Sprinsky (2002) examined affine and similarity (Helmert) least-squares methods, transforming North American Datum of 1927 coordinates into the North American Datum of 1983 (NAD83) system. Coordinate transformations are frequent challenges in mapping, geodesy, surveying engineering, photogrammetry, computer vision, and Geographical Information Science (GIS). These transformations involve converting spatial data (such as maps, orthoimages, and surveyed points) from one coordinate system to another, using a set of control points measured in both coordinate systems to estimate transformation parameters (Felus and Schaffrin, 2005).

Başçiftçi et al. (2010) developed a one-, two-, and three-dimensional transformation program. Their transformations do not alter the physical location of points; only their coordinates are transformed from one system to another. They used linear, quadratic, and cubic methods for 1D transformations, similarity, affine, and projective methods for 2D transformations, and the Bursa-Wolf model for 3D similarity transformations.

Until 2005, the European Datum 1950 (ED50) was Turkey's national coordinate system for various cadastral applications. However, due to crustal movements and displacements, ED50's accuracy became insufficient for many cadastral uses. To address these limitations, Turkey established the Turkish National Fundamental GPS Network (TNFGN), which is based on the International Terrestrial Reference Frame 96 (ITRF-96) coordinate system. This change necessitated the calculation of transformation parameters between the ED50 and ITRF-96 systems (Konakoglu et al., 2016).

A coordinate system is essential to define the location of any feature on Earth. However, coordinate systems vary across countries and regions due to differences in reference surfaces. Commonly used global coordinate systems include ED50, NAD27-83, WGS-84, ITRF, ETRS-89, GCJ-02, and SAD-69. In Turkey, coordinate systems have changed over time to meet needs, with TUD-54, ED50, and ITRF being the primary systems used (Kırıcı and Şişman, 2017).

Datum transformation is required when data from various coordinate systems need to be utilized in a different system. Common transformation methods include similarity, affine, and projective transformations. While similarity transformation is widely used for land coordinates, affine and projective methods are frequently applied in photogrammetry and cartography (Hüsrevoğlu and Tuşat, 2018).

Coordinate transformation is a major concern in geodesy, used to convert coordinates from one datum to another through parameters such as translation terms, scale factors, and rotation angles. The increasing demand for accurate datum transformation in engineering surveys and layout integration has heightened the need for reliable transformation methods. This problem requires calculating transformation parameters using common points with known coordinates in two different datums (Ocalan, 2019).

Shults et al. (2020) explored various approaches for determining transformation parameters, comparing methods like Helmert transformation, bilinear transformation, second- and third-order regression, and fourth-order conformal polynomial transformation within the Almaty city coordinate system. Their findings indicated that none of these methods achieved the desired transformation accuracy, leading them to propose a finite element method (FEM) using Delaunay triangulation.

This paper investigates the two-dimensional similarity (Helmert) and affine transformations. Since geodetic measurements can contain incompatible data, a t-distribution test for discordant measures was applied in this study. The Helmert and Affine transformation methods were used on common points, and the relationship between ED50 and ITRF coordinates was assessed in the Değirmendere and Maçka regions of Trabzon, Turkey. The ED50 and ITRF coordinates for common points in these districts were obtained from the Trabzon Provincial Directorate of Cadastre, and the accuracy of the transformation was tested at additional test points. Since the Maçka region is relatively more mountainous and Değirmendere has a smoother topography, this paper investigates the accuracy of traditional two-dimensional coordinate transformation methods under different conditions. Statistical evaluations were conducted on the differences between ED50 coordinates transformed to ITRF-96 and existing ITRF-96 coordinates at these test points, determining which method yielded better results. In this paper, when comparing the Affine and Similarity transformations, it is seen that the Similarity transformation provides much better results.

Previous research by Evsen (2019) stated in his thesis that the accuracy obtained using the Helmert Transformation method is higher than that of the Affine Transformation method. Hüsrevoğlu and Tuşat (2018) used similarity, affine, and projective transformation methods, and they proposed that the projective transformation method may be more suitable for transforming terrain coordinates. Ünsal (2009) used similarity, affine, and projective transformation methods too, and he mentioned that the average error was similar for similarity and projective transformations, but the affine transformation yielded less accurate results.

2 METHODS

2.1 2D Helmert (Similarity) transformation model

The two-dimensional similarity (Helmert) transformation is a commonly used method in geodetic studies. This transformation preserves the geometric shape of the object, meaning that while the angles between the edges remain constant, the lengths of the sides may scale up or down. In this transformation, four parameters are determined: a scale factor, a rotation angle, and two translations. To calculate these parameters, at least two points with known coordinates are required in both systems. When more than two common points are available, the transformation parameters are estimated using the Least Squares method, a statistical technique for parameter estimation. An illustration of the similarity transformation is shown in Figure 1.

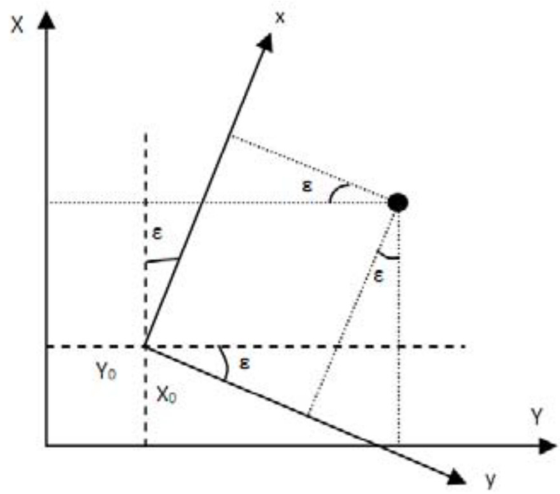


Figure 1. Similarity transformation (Şişman, et al., 2013)

The equations that describe the relationship between the two systems are adapted from Başçiftçi and İnal (2008) and are provided below:

$$X = xm\cos\varepsilon - ym\sin\varepsilon + X_0, \tag{1}$$
$$Y = xm\sin\varepsilon + ym\cos\varepsilon + Y_0, \tag{2}$$

with abbreviations $a = m\cos\varepsilon$, $b = m\sin\varepsilon$, $c = X_0$, $d = Y_0$:

$$X = ax - by + c, \tag{3}$$
$$Y = ay + bx + d, \tag{4}$$

where:

x, y: coordinates in old coordinate system
X, Y: transformed coordinates,
X_0, Y_0: translations elements,
ε: rotation angle between two coordinate systems (ε = arc tan (b / a)),
m: scale factor ($m = \sqrt{a^2 + b^2}$).

If the number of common points is more than two, the transformation parameters are calculated with the Least Squares (Least Squares) method. Functional model and Stochastic Model with matrix representation:

Functional model $\mathbf{v} = \mathbf{A}\,\mathbf{X} - \mathbf{l}$,
Stochastic Model $\mathbf{P} = \mathbf{E}$,
Principle of adjustment [vv]= $\mathbf{v}^T\mathbf{v}$ = min.
Correction equations of coordinates:

$$\begin{aligned} ax_1 - by_1 + c &= X_1 + V_{x_1}, \\ ay_1 + bx_1 + d &= Y_1 + V_{y_1}, \\ &\dots \\ ax_n - by_n + c &= X_n + V_{x_n}, \\ ay_n + bx_n + d &= Y_n + V_{y_n}. \end{aligned} \tag{5}$$

$$\mathbf{A} = \begin{bmatrix} x_1 & -y_1 & 1 & 0 \\ y_1 & x_1 & 0 & 1 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ x_n & -y_n & 1 & 0 \\ y_n & x_n & 0 & 1 \end{bmatrix}_{2nx4} \quad \mathbf{X} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4x1} \quad \mathbf{l} = \begin{bmatrix} X_1 \\ Y_1 \\ \dots \\ \dots \\ X_n \\ Y_n \end{bmatrix}_{2nx1} \quad \mathbf{v} = \begin{bmatrix} V_{X_1} \\ V_{Y_1} \\ \dots \\ \dots \\ V_{X_n} \\ V_{Y_n} \end{bmatrix}_{2nx1}, \tag{6}$$

where:

A: Design (coefficient) matrix,
X: Unknown model parameter vector,
l: Observation vector of control points.

Vector of unknowns is found through the following equations;

$$\mathbf{N} = \mathbf{A}^T\mathbf{P}\mathbf{A}, \tag{7}$$
$$\mathbf{n} = \mathbf{A}^T\mathbf{P}\mathbf{l}, \tag{8}$$
$$\mathbf{X} = \mathbf{N}^{-1}\mathbf{n}. \tag{9}$$

After the unknowns are found:

$$\mathbf{V} = \mathbf{A}\mathbf{X} - \mathbf{l}, \tag{10}$$

the corrections to be brought to the common point coordinates are calculated from the equation (10).

The mean square error (m_0), which represents the precision of the transformation operation, is calculated by equation (11):

$$m_0 = \sqrt{\frac{\mathbf{V}^{\mathrm{T}}\mathbf{P}\mathbf{V}}{2n-u}}, \quad (11)$$

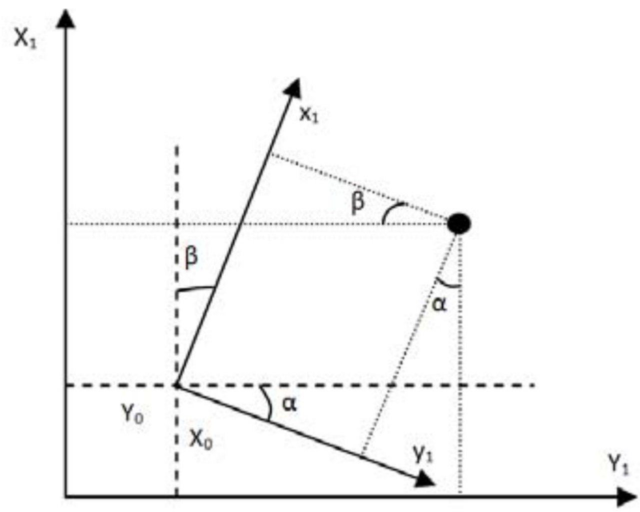
where n is the number of common points, u is the number of unknown new coordinate transformation parameters and $f = 2n - u = 2n - 4$ represents degrees of freedom of the similarity transformation.

The mean errors of the transformation parameters are found using the diagonal elements of the matrix $\mathbf{Q_{xx}}$:

$$(\mathbf{A}^{\mathrm{T}}\mathbf{P}\,\mathbf{A})^{-1} = \mathbf{N}^{-1} = \mathbf{Q_{xx}}, \quad (12)$$
$$m_a = \pm m_0\sqrt{q_{aa}}. \quad (13)$$

2.2 2D Affine transformation model

The two-dimensional affine transformation is primarily used in photogrammetric and cartographic studies. In affine transformation, the coordinate axes are not perpendicular and have different scales along each axis. The scale remains constant in a given direction, but it varies if the direction changes. While parallel lines remain parallel after the transformation, angles between them may change. The affine transformation includes six parameters: two translations, two rotations, and two scale factors. To determine these parameters, at least three common points in both systems are required. When there are more than three common points, the transformation parameters are estimated through an adjustment process using the least squares method. An illustration of the affine transformation is provided in Figure 2.



x_1, y_1: Old coordinate system
X_1,Y_1: Transformed coordinates
a, $ß$: Rotation angles between two coordinate systems
X_0, Y_0: Translations elements
m_x, m_y: Scale factors

Figure 2. Affine transformation (Şişman et al., 2013)

The equations that describe the relationship between the two systems are adapted from Haberler and Kahmen (2003), Şişman et al (2013) are provided below:

$$X_i = m_x x_i \cos\alpha - m_y y_i \sin\beta + X_0, \tag{14}$$
$$Y_i = m_x x_i \sin\alpha + m_y y_i \cos\beta + Y_0. \tag{15}$$

is expressed by the equations (14) and (15). Let's use abbreviations in these equations;

$a = m_x \cos\alpha$, $b = -m_y \sin\beta$, $c = X_0$, $d = m_x \sin\alpha$, $e = m_y \cos\beta$ and $f = Y_0$:

$$X = ax + by + c, \tag{16}$$
$$Y = dx + ey + f. \tag{17}$$

Correction equations of coordinates:

$$\begin{aligned} ax_1 + by_1 + c &= X_1 + V_{x_1}, \\ dx_1 + ey_1 + f &= Y_1 + V_{y_1}, \\ &\dots \\ ax_n + by_n + c &= X_n + V_{x_n}, \\ dx_n + ey_n + f &= Y_n + V_{y_n}. \end{aligned} \tag{18}$$

$$\mathbf{A} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix}_{2nx6} \quad \mathbf{X} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}_{6x1} \quad \mathbf{l} = \begin{bmatrix} X_1 \\ Y_1 \\ \dots \\ \dots \\ X_n \\ Y_n \end{bmatrix}_{2nx1} \quad \mathbf{V} = \begin{bmatrix} V_{X_1} \\ V_{Y_1} \\ \dots \\ \dots \\ V_{X_n} \\ V_{Y_n} \end{bmatrix}_{2nx1}. \tag{19}$$

Vector of unknowns is found through the equation (9).

Scale factors are: $m_x = \sqrt{a^2 + d^2}$ and $m_y = \sqrt{b^2 + e^2}$.

Rotation angles are: α = arc tan (d/a) and β = arc tan ($-b/e$).

Degrees of freedom of the affine transformation is $f = 2n - u = 2n - 6$ (Zhang et al, 2016).

2.3 Determination of Inconsistent Observations

In geodetic measurements conducted for various purposes, it is common to encounter gross errors and inconsistencies. Measurements that contain errors with magnitudes close to random measurement errors can only be detected through an inconsistent measurement test, which is applied as part of the adjustment calculation (Şişman, 2005).

Table 1
Summary chart of inconsistent observation tests

Method	Test Value	Distribution	Critical Value
Baarda(Data-Snooping)	$T_{vi} = \lvert v_i \rvert / (\sigma_0 \sqrt{Q_{v_i v_i}})$	N (0,1) Normal Distribution	$N_{1-\alpha/2}$
Pope (Tau)	$T_{vi} = \lvert v_i \rvert / (m_0 \sqrt{Q_{v_i v_i}})$	τ_f Tau Distribution	$\tau_{f,1-\alpha/2}$
t-test	$T_{vi} = \lvert v_i \rvert / (m_{0i} \sqrt{Q_{v_i v_i}})$	t_{f-1}	$t_{f-1,1-\alpha/2}$

t- Distribution

Data-Snooping (Baarda, 1968), Pope test (Pope, 1976) and t test (Koch,1999) are the classical outlier tests used frequently in geodetic applications. Test values are calculated and compared with the critical value. The t-test was chosen in this study because it allows for detecting outliers by evaluating residuals relative to a recalculated variance that excludes the suspected inconsistent measurement. This recalculation makes the t-test particularly effective when dealing with isolated inconsistencies in otherwise accurate data. No inconsistent measure has been found according to t-test.

In Table 1, Baarda uses the theoretical variance (σ_0^2), Pope uses the estimated variance (m_0^2) found as a result of the adjustment, in the t test is used variance value (m_{0i}^2) calculated after the effect of this inconsistent measure is removed, instead of calculated with the inconsistent measure (s_0^2) (Erol, 2008).

3 STUDY AREA

In this study, Trabzon province in Turkey was selected as the study area. Trabzon is bordered by the Black Sea to the north, Gümüşhane and Bayburt to the south, Rize to the east, and Giresun to the west. Located between 38° 30' and 40° 30' east meridians and 40° 30' and 41° 30' north parallels, Trabzon covers a surface area of 4,664 km². An illustration of Trabzon is provided in Figure 3.

Figure 3. Trabzon province of Turkey (Saygili, 2017)

Maçka and Ortahisar districts in Trabzon province were selected for this study. ED50 and ITRF coordinates of common points within these districts were obtained from the Trabzon Provincial Directorate of Cadastre. Two-dimensional similarity (Helmert) and affine transformation methods were applied to these common points to define the relationship between the two coordinate systems.

In the application conducted in the Değirmendere area of Ortahisar district, 60 common points were selected in both systems, and the accuracy of the transformation was tested using 137 points.

The application was then repeated in Maçka district, where 30 common points were selected, and the transformation accuracy was tested with 76 points. The distribution of test and control points in ED50 and ITRF-96 datums in Maçka and Değirmendere districts is shown in Figures 4, 5, 6, and 7. In selecting the test and control points, care was taken their homogeneous distribution in study area. In the figures, red triangle symbols indicate control points and blue plus symbols indicate test points.

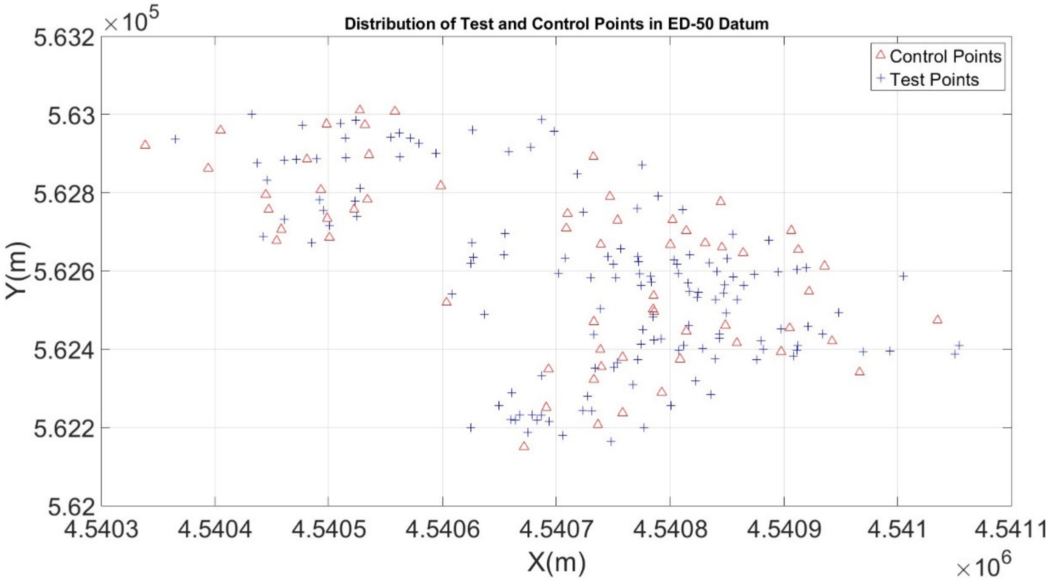


Figure 4. Distribution of control and test points in the Değirmendere in the ED50 datum

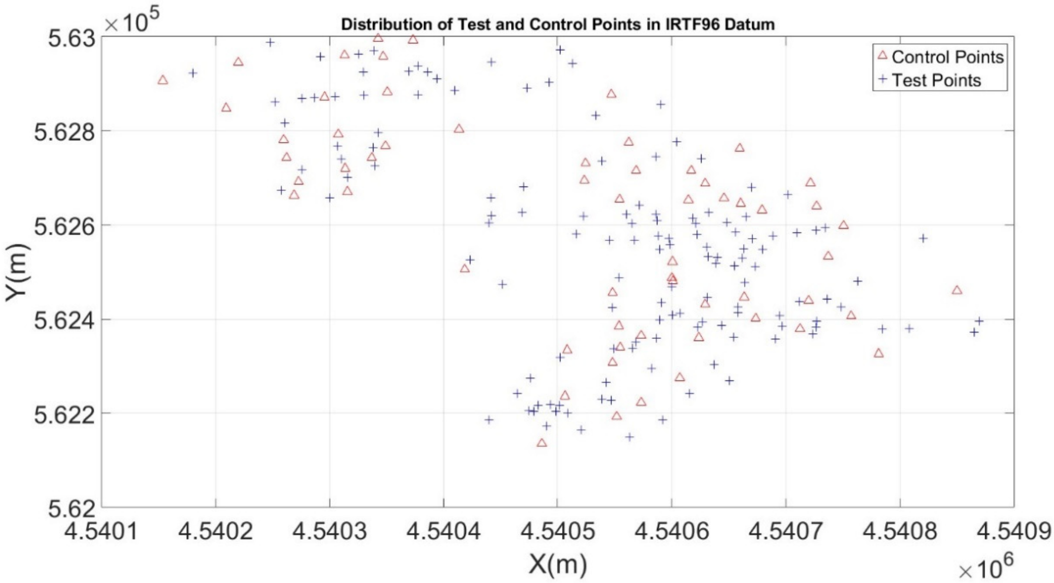


Figure 5. Distribution of control and test points in the Değirmendere in the ITRF-96 datum

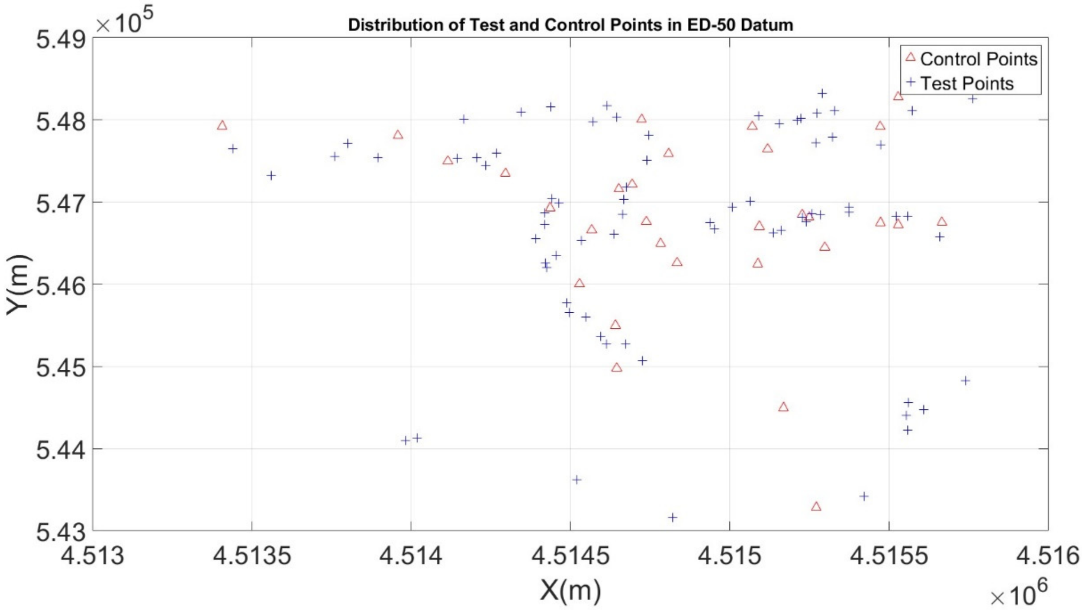


Figure 6. Distribution of control and test points in the Maçka in the ED50 datum

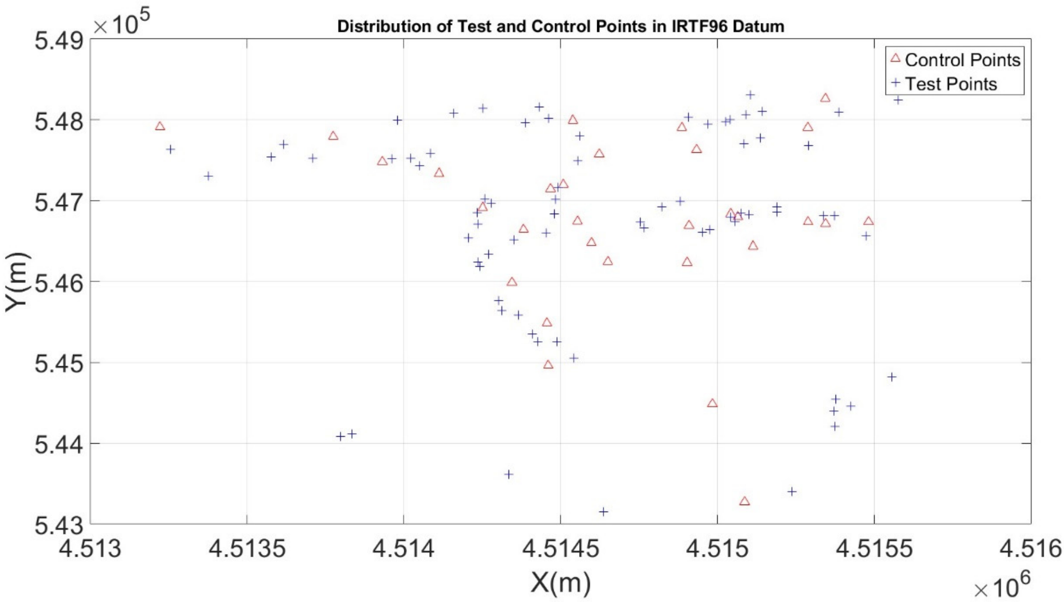


Figure 7. Distribution of control and test points in the Maçka in the ITRF-96 datum

4 RESULTS AND DISCUSSION

4.1 Helmert Transformation's Results

Helmert transformation's parameters and **X** unknown matrix values in Değirmendere and Maçka have been shown in Table 2 and Table 3 respectively.

Table 2
Helmert transformation parameters in Değirmendere and Maçka

Helmert transformation parameters	Değirmendere	Maçka
X_0 (m)	-48.648522	-130.387144
Y_0 (m)	-79.312500	7.403320
m	0.999972	0.999988
Rotation angle (degree)	0.0010201	359.999812

Table 3
X unkowns matrix in Değirmendere and Maçka

X unknowns matrix	Değirmendere	Maçka
a	0.999972	0.999988
b	0.000018	-0.000003
c	-48.648522	-130.387144
d	-79.312500	7.403320

The mean square error (m_0) representing the precision of the transformation operation and average errors of transformation parameters have been shown in Table 4.

Table 4
The mean square error of transformation (m_0) and average errors of transformation parameters (meter)

Districts	The mean square error (m_0)	Average errors of transformation parameters			
Değirmendere	0.067438	1.90×10^{-9}	1.90×10^{-9}	4.16×10^{-16}	4.16×10^{-16}
Maçka	0.001591	6.39×10^{-11}	6.39×10^{-11}	1.40×10^{-17}	1.40×10^{-17}

The coordinates of the test points in the ED50 datum were transformed to the ITRF-96 datum using the calculated transformation parameters. The differences between the transformed coordinates and the existing ITRF-96 coordinates of the test points were then determined. Statistical data, including minimum and maximum errors, root mean square error (*RMSE*), and mean absolute error (*MAE*) values for these differences, were subsequently calculated.

MAE and computed *RMSE* are as follows:

$$MAE = \frac{1}{n}\left|\sum_{i=1}^{n} X_{ITRF96\,(i)} - X_{transformed(i)}\right|, \tag{18}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_{ITRF96(i)} - X_{transformed(i)})^2}. \tag{19}$$

Statistical values are shown in Table 5.

Table 5
Statistical information values for differences between from ED50 to ITRF-96 transformed coordinates and existing ITRF-96 coordinates in test points

	Değirmendere		Maçka	
Statistical values	***X*-direction errors**	***Y*-direction errors**	***X*-direction errors**	***Y*-direction errors**
Min (m)	-0.0901	-0.0282	-0.0010	0.0015
Max (m)	-0.0890	-0.0272	0.0000	0.0026
***MAE* (m)**	0.08961	0.0277	0.0005	0.0021
***RMSE* (m)**	0.08961	0.0277	0.0006	0.0021

4.2 Affine Transformation's Results

Affine transformation's parameters and **X** unknown matrix values in Değirmendere and Maçka have been shown in Table 6 and Table 7 respectively.

Table 6
Affine transformation parameters in Değirmendere and Maçka

Affine transformation parameters	Değirmendere	Maçka
X_0 (m)	-51.000000	-129.562500
Y_0 (m)	-79.750000	7.1562500
m_x	0.999973	0.999987
m_y	0.999972	0.999988
Rotation α (degree)	0.001026	359.999815
Rotation β (degree)	0.000997	359.999813

Table 7
X unkowns matrix in Değirmendere and Maçka

X unknowns matrix	Değirmendere	Maçka
a	0.999973	0.999987
b	-0.000017	0.000003
c	-51.000000	-129.562500
d	0.000018	-0.000003
e	0.999972	0.999988
f	-79.750000	7.156250

The mean square error (m_0) representing the precision of the transformation operation and average errors of transformation parameters have been shown in Table 8.

Table 8
The mean square error of transformation (m_0) and average errors of transformation parameters (meter)

Districts	The mean square error (m_0)	Average errors of transformation parameters					
Değirmendere	0.173986	$1.19x10^{-5}$	$9.61x10^{-5}$	$7.11x10^{-12}$	$1.19x10^{-5}$	$9.61x10^{-5}$	$7.11x10^{-12}$
Maçka	0.022264	$4.50x10^{-7}$	$3.72x10^{-6}$	$4.43x10^{-15}$	$4.50x10^{-7}$	$3.72x10^{-6}$	$4.43x10^{-15}$

The statistical values for the differences between the ED50 to ITRF-96 transformed coordinates and the existing ITRF-96 coordinates at the test points are presented in Table 9.

Table 9
Statistical information values for differences between from ED50 to ITRF-96 transformed coordinates and existing ITRF-96 coordinates in test points

Statistical values	Değirmendere		Maçka	
	X-direction errors	*Y*-direction errors	*X*-direction errors	*Y*-direction errors
Min (m)	0.2377	0.0269	0.0257	-0.0147
Max (m)	0.2388	0.0280	0.0269	-0.0134
***MAE* (m)**	0.2382	0.0275	0.0263	0.0141
***RMSE* (m)**	0.2382	0.0275	0.0263	0.0141

Evsen (2019) stated that the Helmert Transformation method is higher than that of the Affine Transformation method. Hüsrevoğlu and Tuşat (2018) used similarity, affine, and projective transformation methods, and they proposed that the projective transformation method may be more suitable for transforming terrain coordinates. Ünsal (2009) said that the average error was similar for similarity and projective transformations, but the affine transformation yielded less accurate results. Cai and Grafarend (2009) studied two-dimensional Gauss-Krueger coordinates in German geodetic reference system (DHDN) into UTM coordinates in the ETRS89 datum with Helmert transformation and affine transformation method. Papp et al. (2002) used 2D Helmert, affine and 5th order polynomial transformations methods between different datums and they state that the applied methods meet the accuracy requirements for both geodetic and geomatic fields. Kurt (2018) examined commonly used affine and similarity coordinate transformations, highlighting potential errors through an illustrative example. He suggests that similarity transformation should take precedence in the process, followed by the application of similarity and affine transformations in sequence. Finally, he stresses the importance of statistically testing the results and transformation method should be determined according to the results

In future studies, alternative transformation methods could be explored, such as projective transformation methods, Radial Basis Functions (RBF), grid methods, or approaches based on artificial intelligence (AI) and machine learning (ML). Additionally, the number of common points and their locations could be considered as variables, and comparisons could be made accordingly.

4 CONCLUSIONS

This study aimed to determine the 2D transformation parameters needed to convert the ED50 datum to the ITRF-96 datum. For this transformation, the two-dimensional similarity (Helmert) and Affine transformation methods were applied in the Maçka and Ortahisar districts of Trabzon province. Point coordinates (X, Y) that were known in both the ED50 and ITRF-96 datums in the study area were used as data. A total of 197 points were used in Değirmendere, including 60 common points and 137 test points, while in Maçka, 106 points were used, consisting of 30 common points and 76 test points. The two-dimensional similarity (Helmert) and Affine transformation methods were applied to these common points to establish the relationship between the two systems. Additionally, test points's ED50 datum coordinates were transformed to ITRF-96 coordinates using these parameters. The differences between the transformed coordinates and the current coordinates of these test points were analyzed. The primary objective was to evaluate the accuracy of transformation methods and assess regional differences in transformation sensitivity.

According to the Helmert (similarity) transformation results (Table 4), the sensitivity of the transformation (m_0) is approximately 7 cm in Değirmendere and 0.1 cm in Maçka. As shown in Table 4, the sensitivity of the transformation parameters is significantly better in the Maçka region.

- According to the evaluation at the test points (Table 5), the RMSE value in Değirmendere is 9 cm in the x direction and 3 cm in the y direction. In Maçka, the RMSE value is 0 cm in the x direction and 0.2 cm in the y direction.
- According to the Affine transformation results (Table 8), the sensitivity of the transformation (m_0) is about 17 cm in Değirmendere and 2 cm in Maçka. As seen in Table 8, the sensitivity of the transformation parameters is much better in Maçka.
- According to the evaluation at the test points (Table 9), the RMSE value in Değirmendere is 24 cm in the x direction and 3 cm in the y direction. In Maçka, the RMSE value is 3 cm in the x direction and 1 cm in the y direction.
- It has been observed that the sensitivity parameters of the transformation in the Maçka region are better than those in Değirmendere. This may be related to the number of common points, which is fewer in Maçka and the regions's topography. Maçka is relatively more mountainous than Değirmendere.
- When comparing the Affine and Similarity transformations, it is evident that the Similarity transformation provides much better results.
- In future studies, alternative transformation methods can be applied to the study area. Methods such as Radial Basis Functions (RBF), grid methods, and approaches based on artificial intelligence (AI) and machine learning (ML) can be explored and compared to the results of this study.

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