Primljeno / Recived: 11.08.2016.
Prihvaćeno / Accepted: 1.12.2016.

UDK 528.21:27:38/ 530.122/ 681.113.94
Pregledni naučni rad / Review article

IS THE GEOPOTENTIAL DIRECTLY MEASURABLE? (GAUSS, BRUNS, EINSTEIN)

JE LI GEOPOTENCIJAL DIREKTNO MJERLJIV? (GAUS, BRUNS, AJNŠTAJN)

Helmut Moritz

ABSTRACT

It has been pointed out by the great Swedish geodesist Arne Bjerhammar and others around1985 that it is possible to replace the classical method of spirit leveling for determining differences of the geopotential by a much more direct and elegant method, measuring the frequency of atomic clocks. This is impossible by classical physics and requires methods of Einstein's General Theory of Relativity. The principle is that the geopotential can be "felt" by the "proper time" of this theory, but there remained the problem that the measuring accuracies were unthinkably high in 1985 and even later. To get a leveling accuracy of 1 cm, we must measure these frequencies to a relative accuracy of 10-18. Reaching such accuracies provided a great challenge to high-precision time observation all over the world, from USA to China. Now it seems that the required frequency accuracy is being reached.

The author tries to give a short introductory review accessible to geodetic students and surveyors. It is purely didactic.

Key words: geopotential, relativity, atomic clock, spirit leveling, geoid, geopotentiometers.

SAŽETAK

Veliki švedski geodeta Arne Bjerhammar (i neki drugi), istakao je oko 1985. godine, da je klasičnu metodu geometrijskog nivelmana za određivanje razlika geopotencijala moguće zamijeniti mnogo direktnijom i elegantnom metodom, mjerenjem frekvencije atomskih satova. Ovo nije moguće metodama klasične fizike, te zahtijeva primjenu Ajnštajnove Teorije općeg relativiteta. Princip je da se geopotencijal može "osjetiti" pomoću "pravog vremena" ove teorije, ali ostaje problem što je tačnost mjerenja bila nezamisliva u 1985. godini, pa čak i poslije. Da bi se dobila tačnost nivelanja od 1 cm, frekvencije se moraju mjeriti s relativnom tačnošću od 10-18. Dostizanje ove tačnosti bio je ogroman izazov za sve svjetske opservatorije za visokoprecizno mjerenje vremena, od SAD do Kine. Čini se da je zahtijevana tačnost ipak dostignuta.

Autor nastoji dati kratak, potpuno didaktički uvod, pristupačan studentima geodezije i geodetima.

Key words: geopotencijal, relativitet, atomski sat, geometrijski nivelman, geoid, geopotenciometri.

1 PREFACE

Every student of surveying knows the picture of the Earth as a set of level surfaces (or geopotential surfaces) and plumb lines, somewhat as in the following figure, taken from Figure 2-2 in the books (Heiskanen and Moritz, 1967; Heiskanen and Moritz, 2000 or Hofmann-Wellenhof and Moritz, 2005). (The abbreviation PG (Physical Geodesy) denotes any of these books.)

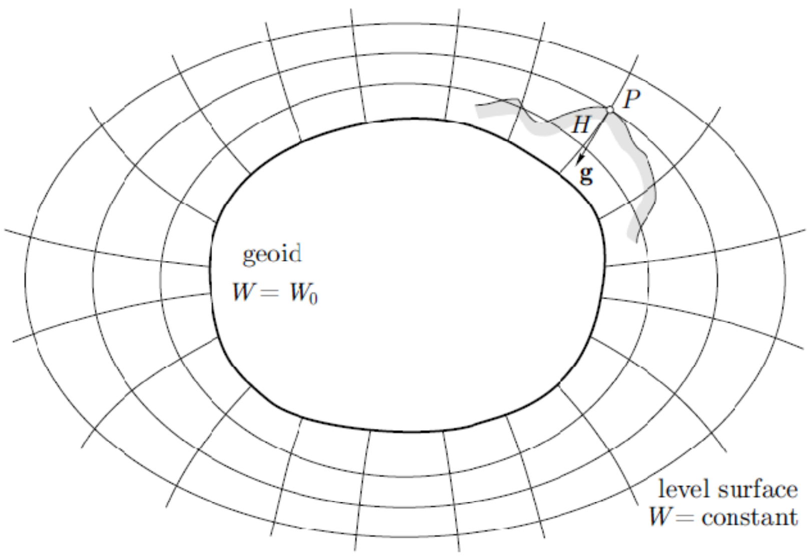

Figure 1. Level surfaces and plumb lines (Hofmann-Wellenhof and Moritz, 2005).

Level surfaces are surfaces of constant geopotential *W*. The simple *spirit leveling* in surveying or civil engineering defines a tiny ("infinitesimal") line on a level surface as a *horizontal surface.* The problem is then to put these tiny lines together, to "*integrate*" these "differential" lines (Heiskanen and Moritz, 1967; section 4.1).

Surveyors hardly like spirit leveling: it is cumbersome, slow and "local" only (some 10 to 50 km) and frequently too expensive. Nevertheless, it is well known and frequently used. So far, it is the only method for determining geopotential differences. It is not possible to"jump" from one end of a leveling line to the other end to measure differences $W_2 - W_1$ directly.

We know, of course, that the *geoid* is a particular level surface, namely the one at sea level, $W = W_0$. This definition of a *mathematical Earth surface* goes back to Carl Friedrich Gauss. An ingenious generalization is due to Heinrich Bruns. Why not define the *Earth's figure* mathematically by the three-dimensional (spatial) function *W(x,y,z)* itself? I quote from PG (end of section 2-2 of Heiskanen and Moritz, 1967; Heiskanen and Moritz, 2000 or Hofmann-Wellenhof and Moritz, 2005): "At first glance the reader is probably perplexed about this definition, which is due to Bruns (1878), but its meaning is easily understood: if the potential *W* is given as a function of the coordinates *x, y, z,* then we know all the level surfaces including the geoid; they are given by the equation:

$$W(x,y,z) = \text{constant}. \quad (1)$$

The geoid is the particular level surface for which the constant has been denoted by W_0.

2 POINTWISE (DIRECT) GEOPOTENTIOMETRY

It is a well-known fact that geopotential differences can be measured along lines, leveling lines. Classically, it is not possible to measure an "absolute" net of geopotential stations of *W* all around the globe. Thus there is not yet a global pointwise net of "*absolute geopotential stations*" (measuring *W*), comparable with a pointwise global net of absolute gravity stations, at least not with classical mechanics. Nature does not "feel" the potential *W*, in the way it feels gravity. Specialists believe, however, that we are not too far from such a global *W-net* ("*Potential Net"),* somewhat similarly to an absolute *g-net* ("*Gravity Net*"), (*W* is the potential of *g*, see: Heiskanen and Moritz, 2000; sec. 2-2).The tedious classical leveling lines over long distances from a principal *W*-point to the next principal *W*-point might soon be a matter of the past.
For measuring *gravity g,* a classical pendulum clock will do; for measuring the geopotential no such sensor has yet been available. A clock, yes, but not a mechanical clock. It must be an *atomic clock,* subject to the laws of the *Einstein's General Theory of Relativity*. This is not even very difficult to understand in principle. Space and Time here are not flat, but curved, although very little.
The curved time of Einstein's theory is called *proper time* (in German: Eigenzeit). Atomic clocks measure proper time. More in detail, "if we associate a coordinate with a space craft, then the time coordinate will be proper time, and an atomic clock placed in the space craft will measure proper time. This appears quite natural because an atom, so to speak, "feels" proper time. It is also in agreement with experimental facts."
These sentences are from the book (Moritz and Hofmann-Wellenhof, 1993).
Here only plausible indications are provided instead of detailed derivations.
The basic formula (5.135) from (Moritz and Hofmann-Wellenhof, 1993) looks even rather innocent:

$$g_{00} = -(1 - 2\, W/c^2). \quad (2)$$

W denotes the usual geopotential, and *c* is the velocity of light (about 300 000 km/sec), and g_{00} is a special coefficient expressing the effect of space-time curvature which is extremely close to -1 (since the light velocity is so high).
This is the mathematical expression of what is known as *relativistic atomic time change*, and is the only way the geopotential affects time, though incredibly little. Furthermore W/c^2 is dimensionless, so we can say: *potential W affects time in this way*, of course with fabulously

accurate coefficients, but nothing else. So to speak, *W can be measured by time,* or even more strongly*, W is time* (in appropriate units).

Of course, the coefficient of *W* is fantastically small, because the light velocity is so large, so the accuracies must be exorbitantly high for this method to work. Among others, it was the great Swedish geodesist Arne Bjerhammar who had this idea in1985, but he considered it not feasible at that time. To get a separation of two neighboring level surfaces of about 1 cm (but over distances of thousands of kilometers, from Hamburg to the Mount Everest!!) one would need a relative frequency accuracy of $10^{-18.}$

Absolutely impossible, until 2 or 3 years ago! Now it is a very hot and realistic research topic. It has the great advantage, that it is related to the problem of *precise global time nets*, which may be even more important than our geodetic problem. Almost every country possessing adequate technology investigates this opportunity.

For references, see the internet (for instance, Wikipedia, *Atomic clock).* A comprehensive monograph for the specialist is Enrico May: *Time, Atomic Clocks, and Relativistic Geodesy*, German Geodetic Commission, A124, München 2013 (Mai, 2013). It contains much literature and is available on the internet.

So the geopotential is measurable directly: *the geopotential W is directly (pointwise) measurable in the relativistic-spacetime* after all! Relativistic spacetime *does feel* the geopotential. Like clocks, yes, *but not mechanical clocks*: It must be an *atomic clock,* subject to the laws of Einstein's *General Theory of Relativity*.

Thus there are instruments, which may be called *geopotentiometers*, which measure *W directly.*

As we have seen, absolute gravimetry and absolute geopotentiometry, if at all possible, are somewhat similar. In a way, they are based on one principle, OSCILLATIONS: the mechanical pendulum and the modern atomic clocks are both oscillations, how much they differ in all other respects.

For a person who understands only a little the General Theory of Relativity, the *theory* of atomic clocks is simple, and it is for such persons that the present little review article has been written. Actually, the book "Geometry, Relativity, Geodesy" has been written to show that the theory of relativity is not so difficult intuitively and fits well to geodetic thinking. Perhaps some additional considerations will help.

3 ATOMIC CLOCKS AND FREQUENCY

There is one concept which has not yet been explained: the concept of *frequency* f .The frequency of an atomic clock in the reciprocal of its *period* or *wavelength* T. Frequency is the number of oscillations per second:

$$f = 1/T \quad \text{and} \quad T = 1/f. \tag{3}$$

Actually, this is basic high-school physics; see, for instance, eq. (5.141) of the book (Moritz and Hofmann-Wellenhof, 1993). A very high frequency of atomic clocks, now often quoted (see also above) would be 10^{18}**,** which is1000000000000000000 (18 zeros!) oscillations per second or in other terms (loosely) an atomic clock which is stable to 1 second over tree billion years. Can you imagine? I cannot, but I believe.

Now we are in a position to understand the following formula, which is a consequence of eqs. (5.140) and (5.141) of the book (Moritz and Hofmann-Wellenhof, 1993), in a slightly different notation:

$$(f_2 - f_1) / f = (W_2 - W_1) / c^2. \tag{4}$$

Take two points P_1 and P_2 anywhere at the Earth's surface (in the above example Hamburg and Mount Everest). To cover the distance by a line of spirit leveling is unfeasible but also unnecessary if we fly from Hamburg to Mount Everest by an aircraft supposed to carry the same atomic clock. In Hamburg we measure frequency f_1 and on Mount Everest we measure f_2 with the highest possible precision.

For f it is sufficient to take an approximate value, say $(f_1 + f_2)/2$. The light velocity c is the usual given constant. What is the conceptual difference to the spirit leveling line mentioned above? *We do not need the line*: we need only to measure f_1 and f_2 at the end points P_1 and P_2 to get potential differences $W_2 - W_1$. Only? Yes, just the difference between two discrete values; there is no leveling line. Exactly like in measuring absolute gravity. (We might soon have absolute geopotentiometry just like we had for along time absolute gravimetry!)

Of course, it is relatively easy to play with formulas and numbers. (So far we have disregarded quantum theory, which also plays a key role in atomic clocks.) The real difficulty is to realize instruments which provide such fantastical accuracies. Here I must leave the reader and refer him/her to the specialist literature.

REFERENCES

Heiskanen, W. A., Moritz, H. (1967). *Physical Geodesy*. San Francisco: Freeman.

Heiskanen, W. A., Moritz, H. (2000). *Fizička geodezija*. Belgrade: Građevinski fakultet Univerziteta u Beogradu, Institut za geodeziju.

Hofmann-Wellenhof, B., Moritz, H. (2005). *Physical Geodesy*, 2nd ed. Wien: Springer.

Ludlow, A. D., Boyd, M. M., Ye, J. (2015, Feb., 10). Optical atomic clocks. Retrieved from https://arxiv.org/pdf/1407.3493v2.

Mai, E. (2013). *Time, Atomic Clocks, and Relativistic Geodesy*. [Report]. München: Deutsche Geodätische Kommission der Bayerischen Akademie der Wissenschaften. Retrieved from https://www.dgk.badw.de/fileadmin/user_upload/Files/DGK/docs/a-124.pdf.

Moritz, H., Hofmann-Wellenhof, B. (1993). *Geometry, Relativity, Geodesy*. Karlsruhe: Wichmann.

NIST Launches a New U.S. Time Standard: NIST-F2 Atomic Clock. (2016, Oct. 14). Retrieved from https://www.nist.gov/news-events/news/2014/04/nist-launches-new-us-time-standard-nist-f2-atomic-clock.

Shen, Z., Shen W. (2015). Geopotential difference determination using optical atomic clocks via coaxial cable time transfer technique and a synthetic test. *Geodesy and Geophysics 6*(5), 344–350. doi: http://dx.doi.org/10.1016/j.geog.2015.05.012.

Author

Helmut Moritz
Professor Dr. techn. Dr.-Ing.E.h.Dr.Sc.h.c.Dr.h.c
Member, Austrian Academy of Sciences
Graz University of Technology
Steyrergasse 30, A-8010 Graz
Austria
E-mail adress: helmut.moritz@tugraz.at